



# Theoretical Studies and App-Development on Algorithms & Visualization on Geodetic and Structural Health Monitoring (SHM) at the Reference Object Television Tower Stuttgart

## Introduction

Geomonitoring, in the domain of Geodesy and Geotechnics, generally deals with various activities involved in the continuous monitoring of civil engineering structures (such as dams, bridges, tunnels), historical and current buildings, during construction or operation, quick alerting during critical events, employing a range of geodetic, geotechnical and geophysical methods.

An integrated 3D geomonitoring (also referred to as "system analysis", "Structural Health Monitoring (SHM)") concept in the GPS-based Online Control and Alarm System (GOCA) R&D (Research and Development) project focuses on the application of the Finite Element Method (FEM) (in both static and dynamic cases) for the combined parameterization of physical and geometric parameters.

Based on a Neumann series of matrices applied to the FEM nodal displacement

$$\mathbf{u}_{FEM} = \mathbf{K}(\mathbf{p}_k)^{-1} \cdot \mathbf{f} \quad (1)$$

(which is the static case of FEM-based SHM where aside the nodal force vector  $\mathbf{f}(t)$  only the stiffness matrix  $\mathbf{K}(\mathbf{p}_k)$  of a parameterization is possible), it is feasible to directly use changes  $\Delta\mathbf{p}_k$  in the physical parameters e.g. lack of stiffness because of material deficiency (refer to Fig. 4.5) to parametrize geodetic displacements  $\mathbf{u}_{geod}$  as

$$\begin{aligned} \mathbf{u}(\Delta\mathbf{p}_k)_{geod} &= \mathbf{u}_{FEM} - \mathbf{K}(\mathbf{p}_k)^{-1} \cdot d\mathbf{K}(\Delta\mathbf{p}_k) \cdot \mathbf{K}(\mathbf{p}_k)^{-1} \cdot \mathbf{f} \\ &= \mathbf{K}(\mathbf{p}_k)^{-1} \cdot (\mathbf{I} - d\mathbf{K}(\Delta\mathbf{p}_k) \cdot \mathbf{K}(\mathbf{p}_k)^{-1}) \cdot \mathbf{f} \end{aligned} \quad (2)$$

The dynamic FEM approaches enable additionally the identification of parameter changes in the damping and the mass matrix  $\mathbf{C}(\mathbf{p}_C)$  and  $\mathbf{M}(\mathbf{p}_M)$  of a structure, where the FEM-based equation for the induced damped vibration of a structure under monitoring is reading

$$\mathbf{K}(\mathbf{p}_K) \cdot \mathbf{u}(t) + \mathbf{C}(\mathbf{p}_C) \cdot \dot{\mathbf{u}}(t) + \mathbf{M}(\mathbf{p}_M) \cdot \ddot{\mathbf{u}}(t) = \mathbf{f}(t) \quad (3)$$

and in the case of the damped eigenvibration of the structure the following is holding

$$\mathbf{K}(\mathbf{p}_K) \cdot \mathbf{u}(t) + \mathbf{C}(\mathbf{p}_C) \cdot \dot{\mathbf{u}}(t) + \mathbf{M}(\mathbf{p}_M) \cdot \ddot{\mathbf{u}}(t) = \mathbf{0} \quad (4)$$

The Stuttgart TV Tower serves as the reference object for this research with respect to the SHM concept as it is being applied in the GOCA R&D project.

## Objectives

This research had two objectives. The first was to produce a research report on the state-of-the-art in SHM involving the use of the FEM and Kalman Filtering (KF). This is to help with improving the structural health monitoring mechanism currently employed by the GOCA system.

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The second objective was to develop a mobile application, for use by the tourists at the Stuttgart TV Tower, which among other incorporated features like weather and map services, accesses the data output of the GOCA system installed at the tower and generates a visually impressive graphical representation of the said data.

## Results

Concerning the first objective, four cases published between 2016 and 2018 were reviewed. Refer to the Algorithm 1, the EGDF (Extended Gillijns and De Moor Filter) algorithm, in one of the cases, for the joint state/input/parameter estimation of linear systems.

$$\begin{aligned} &(1) \text{ Initialization of } \mathbf{z}_{0|0}, \mathbf{P}_{0|0}^z \\ &(2) \text{ Input identification} \\ &\quad \bar{\mathbf{R}}_k = \nabla_z h_k \cdot \mathbf{P}_{k|k-1}^z \cdot (\nabla_z h_k)^T + \mathbf{R}_k \\ &\quad \mathbf{J}_k = (\mathbf{D}_k^T \bar{\mathbf{R}}_k^{-1} \mathbf{D}_k)^{-1} \mathbf{D}_k^T \bar{\mathbf{R}}_k^{-1} \\ &\quad \hat{\mathbf{u}}_k = \mathbf{J}_k (\mathbf{y}_k - h_k(\mathbf{z}_{k|k-1})) \\ &\quad \mathbf{P}_k^u = (\mathbf{D}_k^T \bar{\mathbf{R}}_k^{-1} \mathbf{D}_k)^{-1} \\ &(3) \text{ Measurement update} \\ &\quad \mathbf{K}_k = \mathbf{P}_{k|k-1}^z (\nabla_z h_k)^T \bar{\mathbf{R}}_k^{-1} \\ &\quad \mathbf{z}_{k|k} = \mathbf{z}_{k|k-1} + \mathbf{K}_k (\mathbf{y}_k - h_k(\mathbf{z}_{k|k-1}) - \mathbf{D}_k \hat{\mathbf{u}}_k) \\ &\quad \mathbf{P}_{k|k}^z = \mathbf{P}_{k|k-1}^z - \mathbf{K}_k (\bar{\mathbf{R}}_k - \mathbf{D}_k \mathbf{P}_k^u \mathbf{D}_k^T) \mathbf{K}_k^T \\ &\quad \mathbf{P}_k^{zu} = (\mathbf{P}_{k|k}^z)^T = -\mathbf{K}_k \mathbf{D}_k \mathbf{P}_k^u \\ &(4) \text{ Time update} \\ &\quad \mathbf{z}_{k+1|k} = f_k(\mathbf{z}_{k|k}, \hat{\mathbf{u}}_k) \\ &\quad \mathbf{P}_{k+1|k}^z \equiv E(\tilde{\mathbf{z}}_{k+1|k} \tilde{\mathbf{z}}_{k+1|k}^T) = \begin{bmatrix} \nabla_z f_k & \nabla_u f_k \\ \mathbf{P}_{k|k}^z & \mathbf{P}_k^{zu} \\ \mathbf{P}_k^{zu} & \mathbf{P}_k^u \end{bmatrix} + \mathbf{G}_k \end{aligned}$$

Algorithm 1: The algorithm of EGDF (Wan et al., 2018).

Though all the cases examined made attempts at providing new means of performing FEM-based SHM, one was most promising because it considered also the need to handle condensed matrices in respect to the resolution of the frequencies. Generally all approaches were missing the SHM-parametrisation - at least of the K- and C-matrices - in respect to the "unhealthiness" of the parameters and algorithms for their detection. Furthermore, the temperature dependencies over a daily and yearly cycle are also not regarded yet;

Going forward independent simulation studies and the SHM-modelling itself should be performed to ascertain the feasibility and effectiveness of the different approaches. A real project, like the Stuttgart tower can be used to collect real data, to do experimental developments and to make a proof of concept (POC) on the different approaches.

Fig. 1 shows some images of the app developed presenting the main page and the visualisations of the position-time series and the motion of tower graphs. In its full dynamic state and when finally installed at the Tower the app would access and visualize in real-time the FIN file data output of the GOCA system.



Fig.1: The main page, position-time series and graph of tower motion visualisations of the app